Film Cooling Flow Loss Model

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1 Cooling mass flow calculation for one blade row

Figure 1 shows the film-cooling model at the blade surface. The cooling gas flows into the blade at T_{ci}, and gets heated internally before issuing from the blade holes at T_{co}. The incoming hot gas at total temperature T_g is entrained into the film, and loses heat into the blade.

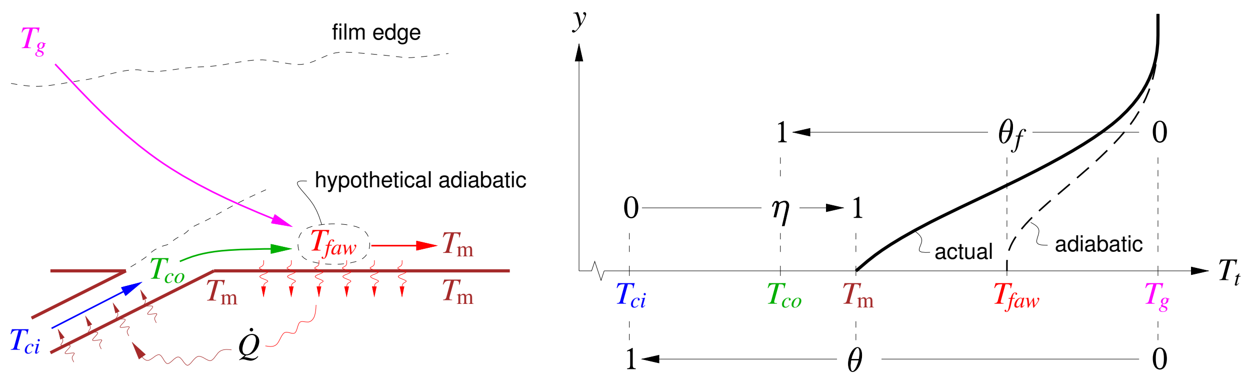

Figure 1: Stream mixing, heat flow, and temperature profiles in film-cooling flow.

As analyzed by Horlock et al [1], the metal temperature $T_{\rm m}$ is characterized by the cooling effectiveness ratio θ,

$$\theta \;\equiv\; \frac{T_r - T_{\rm m}}{T_g - T_{ci}} \;\simeq\; \frac{T_g - T_{\rm m}}{T_g - T_{ci}} \tag{1}$$

where T_r is the hot gas recovery temperature, $T_{\rm m}$ is the metal temperature, T_g is the hot gas inflow total temperature, and T_{ci} is the cooling-air inflow total temperature. The second approximate form in (1) makes the conservative assumption of full temperature recovery.

Since the cooling outlet holes cover only a fraction of the blade surface, the film fluid is a mixture of the cooling-fluid jets issuing at T_{co} and the entrained hot gas at T_g. In the adiabatic (insulated wall) case, this temperature would be some T_{faw}, which is defined in terms of a film-effectiveness factor.

$$\theta_f \;\equiv\; \frac{T_g - T_{faw}}{T_g - T_{co}} \;\simeq\; 0.4 \tag{2}$$

The limiting cases would be

i) $\theta_f = 0$ or $T_{faw} = T_g$ if the cooling-fluid holes are absent, and

ii) $\theta_f = 1$ or $T_{faw} = T_{co}$ if the cooling-fluid holes completely cover the blade.

The experiments of Sargison et al [2] show that the $\theta_f \simeq 0.4$ value is a reasonable surface average for a typical blade.

The cooling efficiency

$$\eta \equiv \frac{T_{co} - T_{ci}}{T_{\rm m} - T_{ci}} = 1 - \exp\!\left(-\frac{A_{cs}}{A_c}\, St_c\right) \simeq 0.7 \tag{3}$$

indicates how much heat the cooling air has absorbed relative to the maximum possible amount before exiting the blade at temperature T_{co}. Horlock et al [1] indicate that for common internal heat transfer/flow area ratios A_{cs}/A_c and Stanton numbers St_c, the $\eta \simeq 0.7$ value is typical. This can be increased somewhat to reflect better cooling flowpath technology (e.g. improved pins, impingement, etc). However, increasing η closer to unity will also incur more total-pressure losses in the cooling flow, so $\eta < 1$ is clearly optimum from overall engine performance.

As indicated by Figure 1, the outer-surface heat inflow from the film must be balanced by the internal heat outflow into the cooling flow. Equating these gives

$$\dot{Q} = A_{sg}\, St_g\, \rho_g V_g\, c_{p_g} (T_{faw} - T_{\rm m}) = \dot{m}_{c_1}\, c_{p_c}\, (T_{co} - T_{ci}) \tag{4}$$

$$A_{sg}\, St_g\, \frac{\dot{m}_g}{A_g}\, c_{p_g} (T_{faw} - T_{\rm m}) = \dot{m}_{c_1}\, c_{p_c}\, (T_{co} - T_{ci}) \tag{5}$$

where A_{sg} is the heat-transfer area of the hot gas, A_g is the flow area of the hot gas, and St_g is the external Stanton number. We now define the cooling/total mass flow ratio for one blade row,

$$\varepsilon_1 \equiv \frac{\dot{m}_{c_1}}{\dot{m}} = \frac{\dot{m}_{c_1}}{\dot{m}_g + \dot{m}_{c_1}} \tag{6}$$

so that equation (4) becomes

$$\frac{\dot{m}_{c_1}}{\dot{m}_g} = \frac{\varepsilon_1}{1 - \varepsilon_1} = St_{\scriptscriptstyle A}\, \frac{T_{faw} - T_{\rm m}}{T_{co} - T_{ci}} \tag{7}$$

$$St_{\scriptscriptstyle A} \equiv \frac{c_{p_g}}{c_{p_c}}\, \frac{A_{sg}}{A_g}\, St_g \simeq 0.035 \tag{8}$$

where $St_{\scriptscriptstyle A}$ is a heat transfer area-scaled Stanton number. Horlock et al [1] argue that for typical blade solidities and aspect ratios, the assumed value $St_{\scriptscriptstyle A} \simeq 0.035$ is reasonable. This will typically need to be increased by a substantial safety factor of 2 or more to allow for parameter uncertainties, hotspots, etc. Improved cooling design would be represented by a decreased safety factor.

Using θ, θ_f, and η to eliminate $T_{\rm m}$, T_{faw}, and T_{co} from (7) gives the following relation between all the dimensionless parameters.

$$\frac{\dot{m}_{c_1}}{\dot{m}_g} = \frac{\varepsilon_1}{1 - \varepsilon_1} = St_{\scriptscriptstyle A}\, \frac{\theta(1 - \eta\, \theta_f) - \theta_f (1 - \eta)}{\eta\, (1 - \theta)} \tag{9}$$

1.0.1 Design case

The design problem is to determine the cooling flow required to achieve a specified $T_{\rm m}$ at the maximum design T_g (e.g. $T_g = T_{t\,4}$ at the takeoff case). Since T_{ci} is also known for any operating point (e.g. $T_{ci} = T_{t3}$), then θ is fully determined from its definition (1). Equation (9) can then be solved for the required design cooling flow ratio for the blade row.

$$\varepsilon_1 \;=\; \left[1 \,+\, \frac{1}{St_A} \, \frac{\eta \, (1-\theta)}{\theta(1-\eta\, \theta_f) \,-\, \theta_f(1-\eta)} \right]^{-1} \tag{10}$$

Figure 2 shows ε_1 versus θ for three scaled Stanton numbers.

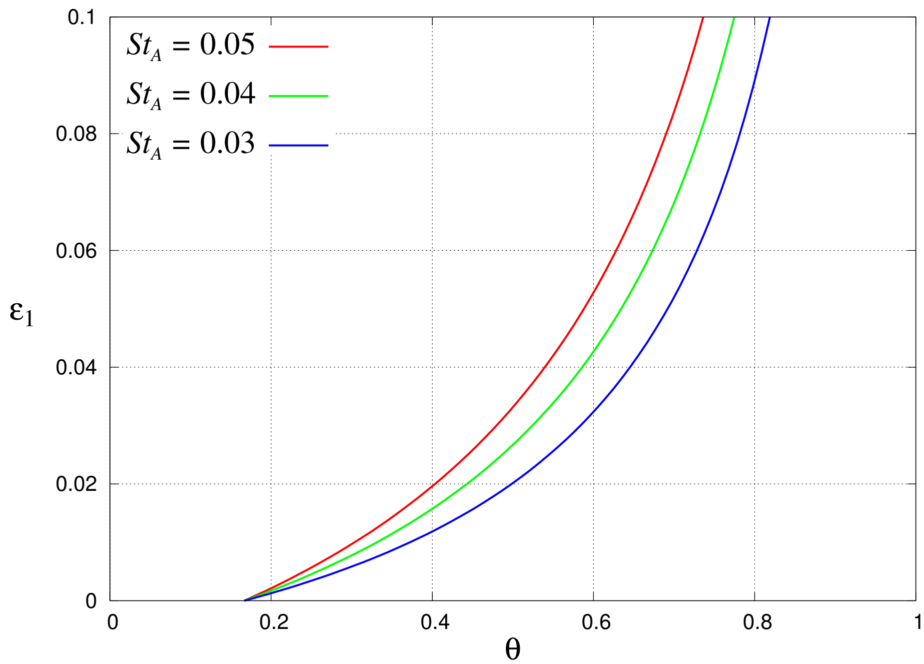

Figure 2: Cooling mass flow ratio ε_1 for one blade row, versus cooling effectiveness θ and scaled Stanton number St_A. Fixed parameters: $\eta = 0.7$, $\theta_f = 0.4$.

1.0.2 Off-design case

If the cooling mass flow is unregulated, it's reasonable to assume that ε_1 will not change at off-design operation if the pressure ratios in the engine do not change appreciably. In that case, θ will not change either, and T_{m} can then be obtained from (1) for any specified T_g and T_{ci}. If the cooling flow ratio ε_1 does change for whatever reason, it's then of interest to determine the resulting metal temperature. Hence, we now solve equation (9) for the new resulting θ in terms of a specified new ε_1.

$$\theta \;=\; \frac{\varepsilon_1 \,\eta \;+\; St_A \,\theta_f (1-\eta)(1-\varepsilon_1)}{\varepsilon_1 \,\eta \;+\; St_A \,(1-\eta\, \theta_f)(1-\varepsilon_1)} \tag{11}$$

2 Total Cooling Flow Calculation

The cooling flow calculations will assume the following quantities are specified, or known from other (e.g. compressor, combustor) calculations:

h_{fuel}	fuel heating value
$T_{t\,\text{f}}$	fuel total temperature
$T_{t\,3}$	compressor exit total temperature
$p_{t\,3}$	compressor exit total pressure
$T_{t\,4}$	turbine inlet total temperature
$p_{t\,4}$	turbine inlet total pressure ($= p_{t\,3}\pi_{\text{b}}$)
T_{m}	metal temperature (for design case)
γ_t	gas specific heat ratio
ε_{c}	total cooling-flow bypass mass flow fraction (for off-design case)
St_A	area-weighted external Stanton number
M_{exit}	turbine blade-row exit Mach number
M_{4a}	representative Mach number at start of mixing zone
$r_{u_{\text{c}}}$	cooling-flow velocity ratio ($= u_{\text{c}}/u_{4a}$)

The cooling/compressor mass flow fraction to be determined,

$$\varepsilon_{\text{c}} \equiv \frac{\dot{m}_{\text{cool}}}{\dot{m}} \tag{12}$$

is defined as the total over all the blade rows which receive cooling flow.

To estimate the blade-relative hot-gas total temperature T_g incoming into each blade row, it is assumed that the inlet Mach number for that blade row is neglible. Hence, the inlet total temperature for a blade row is the same as the static exit temperature of the upstream blade row.

$$T_g \simeq (T_{\text{exit}})_{\text{upstream}} \tag{13}$$

Specifying the burner exit temperature $T_{t\,4}$ and a typical blade-relative exit Mach number M_{exit}, and assuming a small absolute-frame exit Mach number, is then sufficient to determine the blade-relative hot-gas temperatures $T_{g1}, T_{g2}, T_{g3} \ldots$ for all downstream blade rows.

$$T_{g1} = T_{t\,4} + \Delta T_{\text{streak}} \tag{14}$$

$$T_{g2} = T_{t\,4}\left(1 + \frac{\gamma_t - 1}{2} M_{\text{exit}}^2\right)^{-1} \tag{15}$$

$$T_{g3} = T_{t\,4}\left(1 + \frac{\gamma_t - 1}{2} M_{\text{exit}}^2\right)^{-2} \tag{16}$$

$$\vdots$$

The added ΔT_{streak} for T_{g1} is a hot-streak temperature allowance for the first IGV row. According to Koff [3], assuming $\Delta T_{\text{streak}} \simeq 200°\text{K}$ is realistic.

With the row T_g's defined, relation (1) gives the required cooling effectiveness ratio for each blade row,

$$\theta_1 = \frac{T_{g1} - T_{\text{m}}}{T_{g1} - T_{ci}} \tag{17}$$

$$\theta_2 = \frac{T_{g2} - T_{\text{m}}}{T_{g2} - T_{ci}} \tag{18}$$

$$\vdots$$

and relation (11) gives the corresponding cooling mass flow $\varepsilon_1, \varepsilon_2, \varepsilon_3 \ldots$ for each blade row. These are computed until $\varepsilon_{()} < 0$ is reached, indicating that cooling is no longer required. The total cooling mass flow ratio is then the sum of the individual blade-row mass flow ratios.

$$\varepsilon_c \;=\; \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \ldots \tag{19}$$

3 Mixed-out Flow and Loss Calculation

3.1 Loss Model Assumptions

The return of cooling air into the flowpath reduces the total pressure and total temperature seen by the turbine. The most accurate approach is to perform cooling-flow, mixing-loss, and rotor-work calculations separately for each cooled stator and rotor blade row.

To avoid this complication, a simplified model will be used here: The cooling air for all blade rows is assumed to be discharged entirely over the first IGV, and to fully mix out before the flow enters the first turbine rotor. This seems to be a reasonable simplification given that the first IGV blade row typically requires the bulk of the cooling flow. A resulting advantage is that now there is no need to perform work calculations for individual turbine stages, greatly simplifying the matching of the turbine with the compressor.

3.2 Loss Calculation

Figure 3 shows the combustor air and cooling mass flow paths assumed for the mixed-out state calculation. The cooling air is assumed to be bled off at the compressor exit, which defines the cooling-flow total temperature.

$$T_{ci} \;=\; T_{t\,3} \tag{20}$$

The cooling air is assumed to re-enter the flowpath over the first IGV.

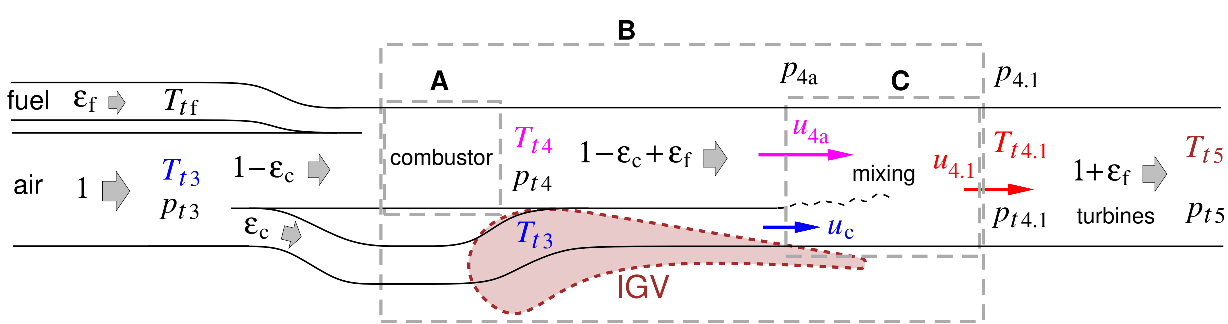

Figure 3: Combustor and film-cooling flows, with mixing over and downstream of IGV. Indicated mass flow fractions are relative to the compressor air mass flow $\dot{m}$. Dashed rectangles are control volumes.

The heat flow $\dot{Q}$ from the air flow to the metal and then to the cooling flow does not need to be considered here, since this heat flow is purely internal to the control volume spanning stations 4 and 4.1, for example.

The cooling flow is assumed to remain unmixed until some representative station 4a (e.g. somewhere between stagnation and IGV exit Mach), where it has velocity $u_{\rm c} = r_{u_{\rm c}} u_{4a}$, and the local static pressure p_{4a}. The mixing then occurs between 4a and 4.1, producing a total-pressure drop and ultimately resulting in a reduced core-flow exhaust velocity.

For clarity and convenience, the equations shown assume a constant average specific heat $\bar{c}_p$. In practice, the calculations would be performed using their equivalent variable–c_p forms.

The heat-balance equation is applied to control volume A in Figure 3, which gives the fuel/compressor-air mass flow ratio $\varepsilon_{\rm f} \equiv \dot{m}_{\rm fuel}/\dot{m}$.

$$h_{\rm fuel}\,\varepsilon_{\rm f}\,\dot{m} \;=\; \bar{c}_p\,(T_{t\,4} - T_{t\,3})\,(1-\varepsilon)\,\dot{m} \;+\; \bar{c}_p\,(T_{t\,4} - T_{t\,\rm f})\,\varepsilon_{\rm f}\,\dot{m} \tag{21}$$

$$\varepsilon_{\rm f} \;=\; \frac{\bar{c}_p\,(T_{t\,4} - T_{t\,3})\,(1-\varepsilon)}{h_{\rm fuel} \;-\; \bar{c}_p\,(T_{t\,4} - T_{t\,\rm f})} \tag{22}$$

The heat-balance equation is next applied to control volume B, which gives the mixed-out total temperature $T_{t\,4.1}$.

$$h_{\rm fuel}\,\varepsilon_{\rm f}\,\dot{m} \;=\; \bar{c}_p\,(T_{t\,4.1} - T_{t\,3})\;\dot{m} \;+\; \bar{c}_p\,(T_{t\,4.1} - T_{t\,\rm f})\,\varepsilon_{\rm f}\,\dot{m} \tag{23}$$

$$T_{t\,4.1} \;=\; \frac{h_{\rm fuel}\,\varepsilon_{\rm f}/\bar{c}_p + T_{t\,3} + T_{t\,\rm f}\varepsilon_{\rm f}}{1+\varepsilon_{\rm f}} \tag{24}$$

The air and cooling flow velocities $u_{4a}, u_{\rm c}$ are obtained from the specified M_{4a} and the specified velocity ratio $r_{u_{\rm c}}$.

$$u_{4a} \;=\; \frac{M_{4a}}{\sqrt{1+\frac{\gamma-1}{2}M_{4a}^2}}\,\sqrt{\gamma R T_{t\,4}} \tag{25}$$

$$u_{\rm c} \;=\; r_{u_{\rm c}} u_{4a} \tag{26}$$

Neglecting any static pressure rise from the mixing, a momentum balance applied to control volume C gives the mixed-out velocity $u_{4.1}$.

$$p_{4.1} \;\simeq\; p_{4a} \;=\; p_{t\,4}\left(1+\frac{\gamma-1}{2}M_{4a}^2\right)^{-\gamma/(\gamma-1)} \tag{27}$$

$$(1+\varepsilon_{\rm f})\,\dot{m}\,u_{4.1} \;=\; (1-\varepsilon_{\rm c}+\varepsilon_{\rm f})\,\dot{m}\,u_{4a} \;+\; \varepsilon_{\rm c}\,\dot{m}\,u_{\rm c} \tag{28}$$

$$u_{4.1} \;=\; u_{4a}\,\frac{(1-\varepsilon_{\rm c}+\varepsilon_{\rm f})\,u_{4a} \;+\; \varepsilon_{\rm c}\,u_{\rm c}}{1+\varepsilon_{\rm f}} \tag{29}$$

The mixed-out static temperature and total pressure then follow.

$$T_{4.1} \;=\; T_{t\,4.1} \;-\; \frac{1}{2}\frac{u_{4.1}^2}{\bar{c}_p} \tag{30}$$

$$p_{t\,4.1} \;=\; p_{4.1}\left(\frac{T_{t\,4.1}}{T_{4.1}}\right)^{\gamma/(\gamma-1)} \tag{31}$$

These can now be used as effective turbine inlet conditions for turbine work and turbine pressure-drop calculations.

References

[1] J.H. Horlock, D.T. Watson, and T.V. Jones. Limitations on gas turbine performance imposed by large turbine cooling flows. *Journal of Engineering for Gas Turbines and Power*, 123:487–494, Jul 2001.

[2] J.E. Sargison, S.M. Guo, M.L.G. Oldfield, G.D. Lock, and A.J. Rawlinson. A converging slot-hole film-cooling geometry — part 2: Transonic nozzle guide vane heat transfer and loss. *Journal of Turbomachinery*, 124:461–471, July 2002.

[3] B.L. Koff. Gas turbine technology evolution: A designer's perspective. *Journal of Propusion and Power*, 20(4):577–595, Jul–Aug 2004.