Turbofan Weight Model from Historical Data

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Nomenclature

$b_{()}$	weight-model exponents	$\mathcal{P}_k$	weight-model constants $\{b_{()}\,,\ W_{()}\}$
$W_{()}$	weight-model coefficients	$\mathcal{F}$	least-squares penalty function
$\mathcal{W}$	bare weight function	$\mathcal{R}_k$	residual vector $(=\partial\mathcal{F}/\partial P_k)$
$\dot{m}$	core mass flow at standard conditions	$\mathcal{J}_{kl}$	Jacobian matrix $(=\partial\mathcal{R}_k/\partial P_l)$
α	bypass ratio (*BPR*)	$(\,)_i$	engine-model index
π_o	overall pressure ratio (*OPR*)	$(\,)_k$	weight-model constant index

1 Assumed bare weight form

The dry bare weight of a turbofan engine is assumed to depend on the primary engine parameters as follows.

$$W_{\rm bare} \;=\; \mathcal{W}(\dot{m},\pi_o,\alpha\,;\mathcal{P}_k) \;=\; \left(\frac{\dot{m}}{100\,{\rm lbm/s}}\right)^{b_m}\left[W_0 \;+\; W_\pi\left(\frac{\pi_o}{30}\right)^{b_\pi} + \; W_\alpha\left(\frac{\alpha}{5}\right)^{b_\alpha}\right] \tag{1}$$

$$\mathcal{P}_k \;\equiv\; \{\, b_m\,,\ b_\pi\,,\ b_\alpha\,,\ W_0\,,\ W_\pi\,,\ W_\alpha\,\} \tag{2}$$

$\mathcal{P}_k$ are the six data-fit constants to be determined.

The form (1) is constructed so that W_0 captures the basic weight of the core, W_π captures the added weight which scales with the overall pressure ratio (core length, casing weight, etc), and W_α captures the added weight which scales with the bypass ratio which quantifies the frontal area. W_α also includes the weight of the low-pressure turbine which drives the fan, and the shaft, bearings, etc. of the entire fan spool. The exponents on each term are intended to capture nonlinearities, possibly from the cube-square law and related effects.

The various constant numbers in (1) are baseline values, representative of a CFM56 class engine[1]. These constants could be absorbed into the W_0, W_π, W_α coefficients, but are left out so that the coefficients roughly indicate the weight contributions of the core, the overall pressure ratio, and the fan spool.

2 Model calibration

The six constants $\mathcal{P}_k$ will be determined by nonlinear data fitting from a large number of existing engines. With i denoting the engine-model index, we use the known engine data

$$W_i \;\;,\quad \dot{m}_i \;\;,\quad \pi_{o_i} \;\;,\quad \alpha_i$$

to minimize the least-square penalty function $\mathcal{F}$,

$$\mathcal{W}_i(\mathcal{P}_k) \;\;\equiv\;\; \mathcal{W}(\dot{m}_i,\pi_{o_i},\alpha_i\,;\,\mathcal{P}_k) \tag{3}$$

$$\mathcal{F}(\mathcal{P}_k) \;\;\equiv\;\; \sum_i \frac{1}{2}\left(\frac{\mathcal{W}_i}{W_i}-1\right)^2 r_i \tag{4}$$

where r_i is an arbitrary weighting factor for that engine model.

3 Least-squares system solution

The six unknown parameters are determined by setting the $\mathcal{F}$-extremum derivatives to zero. The resulting equations are nonlinear, and will be solved by the Newton method.

It's convenient to first define the partial derivatives of $\mathcal{F}$ with respect to the six parameters.

$$\bar{m} \equiv \left(\frac{\dot{m}}{100\,\mathrm{lbm/s}}\right)^{b_m} \tag{5}$$

$$\frac{\partial \mathcal{W}}{\partial \mathcal{P}_1} = \frac{\partial \mathcal{W}}{\partial b_m} = \mathcal{W}\,\ln\!\left(\frac{\dot{m}}{100\,\mathrm{lbm/s}}\right) \tag{6}$$

$$\frac{\partial \mathcal{W}}{\partial \mathcal{P}_2} = \frac{\partial \mathcal{W}}{\partial b_\pi} = \bar{m}\, W_\pi \left(\frac{\pi_o}{30}\right)^{b_\pi} \ln\!\left(\frac{\pi_o}{30}\right) \tag{7}$$

$$\frac{\partial \mathcal{W}}{\partial \mathcal{P}_3} = \frac{\partial \mathcal{W}}{\partial b_\alpha} = \bar{m}\, W_\pi \left(\frac{\alpha}{5}\right)^{b_\alpha} \ln\!\left(\frac{\alpha}{5}\right) \tag{8}$$

$$\frac{\partial \mathcal{W}}{\partial \mathcal{P}_4} = \frac{\partial \mathcal{W}}{\partial W_0} = \bar{m} \tag{9}$$

$$\frac{\partial \mathcal{W}}{\partial \mathcal{P}_5} = \frac{\partial \mathcal{W}}{\partial W_\pi} = \bar{m}\left(\frac{\pi_o}{30}\right)^{b_\pi} \tag{10}$$

$$\frac{\partial \mathcal{W}}{\partial \mathcal{P}_6} = \frac{\partial \mathcal{W}}{\partial W_\alpha} = \bar{m}\left(\frac{\alpha}{5}\right)^{b_\alpha} \tag{11}$$

The 6 Newton residuals and the associated approximate 6×6 Jacobian matrix can then be defined by summing over all engine models.

$$\mathcal{R}_k \equiv \frac{\partial \mathcal{F}}{\partial \mathcal{P}_k} = \sum_i \left(\frac{\mathcal{W}_i}{W_i} - 1\right) \frac{\partial \mathcal{W}_i}{\partial \mathcal{P}_k}\,\frac{1}{W_i}\, r_i \tag{12}$$

$$\mathcal{J}_{kl} \equiv \frac{\partial \mathcal{R}_k}{\partial \mathcal{P}_l} = \sum_i \frac{\partial \mathcal{W}_i}{\partial \mathcal{P}_k}\,\frac{\partial \mathcal{W}_i}{\partial \mathcal{P}_l}\,\frac{1}{W_i^2}\, r_i \tag{13}$$

Starting from some initial-guess $\mathcal{P}_k^0$, the Newton system is constructed and solved for the fit-constant changes $\delta\mathcal{P}_k$, which are then used to obtain the next guess $\mathcal{P}_k^{n+1}$.

$$\mathcal{J}_{kl}\,\delta\mathcal{P}_l = -\mathcal{R}_k \tag{14}$$

$$\mathcal{P}_k^{n+1} = \mathcal{P}_k^n + \delta\mathcal{P}_k \tag{15}$$

Operations (12)–(15) are now repeated until $|\mathcal{R}_k|$ drop below some tolerance. A suitable measure of the resulting data-fit error $\mathcal{E}$ is

$$\mathcal{E} = \sqrt{\frac{2\mathcal{F}}{\sum_i r_i}} \tag{16}$$

which in effect is a weighted standard deviation.

Provided that $\mathcal{J}_{kl}$ is reasonably well conditioned, convergence of the iterated sequence (12)–(15) is very rapid. An ill-conditioned system will result if the engine data does not sufficiently span the influence of one or more of the components in $\mathcal{P}_k$.

4 Computed data fits

Public data for about 40 civil turbofan engines has been used to compute the fit constants shown in Table 1. Note that the condition number is extremely large, indicating that the system is quite ill-conditioned, even though the least-squares fit error is reasonable, as can also be seen in Figure 1. The ill-conditioning is either due to some of the constants in $\mathcal{P}_k$ being nearly linearly dependent, or due to them having little influence on $\mathcal{R}_k$.

Table 1: Fit constants for turbofan engines. Condition number $C = 87380$.

b_m	b_π	b_α	W_0	W_π	W_α
0.97056	1.05264	1.28604	1580.6 lb	375.0 lb	1478.8 lb

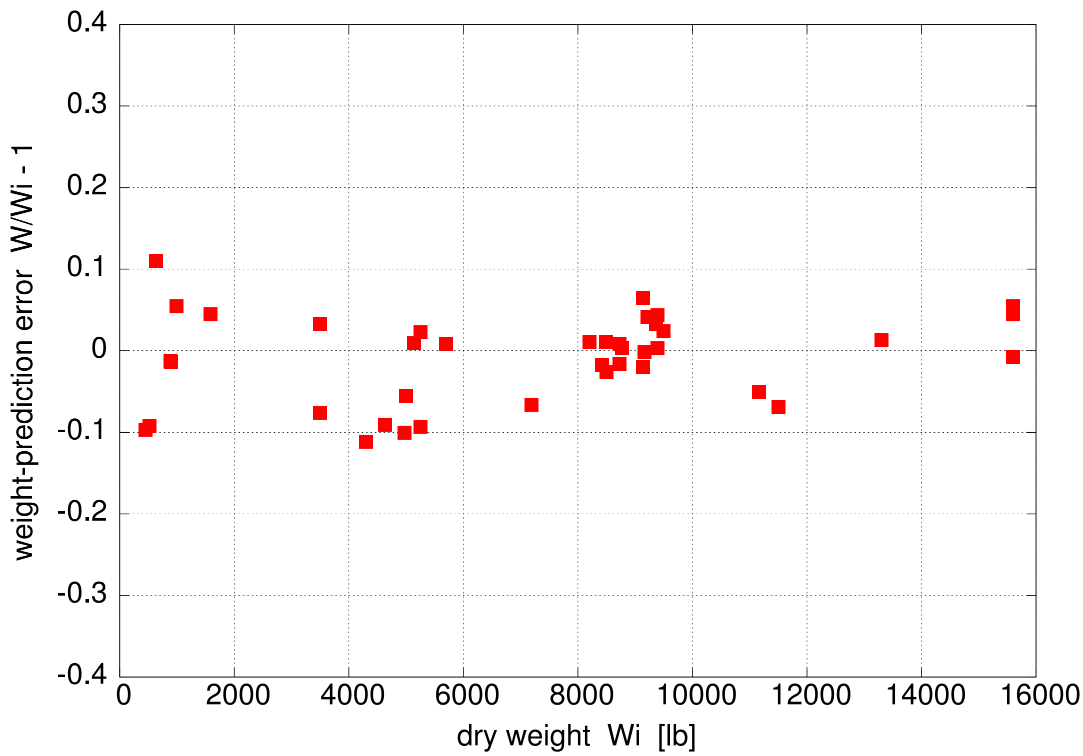

Figure 1: Weight-prediction error for sampled engines, versus engine weight. Fitting error $\mathcal{E} = 0.0540$.

For the case in Table 1 and Figure 1, the ill-conditioning mainly originates in the exponents $b_{()}$ having nearly the same effect on $\mathcal{W}$ as the coefficients $W_{()}$, at the sampled engine parameter values. A simple fix is to freeze the exponents at some reasonable values, by redefining their residuals and associated Jacobians for $k=1,2,3$.

$$\mathcal{R}_1 = b_m - 1.0 \tag{17}$$

$$\mathcal{R}_2 = b_\pi - 1.0 \tag{18}$$

$$\mathcal{R}_3 = b_\alpha - 1.2 \tag{19}$$

$$\mathcal{J}_{k\,l} = \begin{cases} 1 \ , & k = l \\ 0 \ , & k \neq l \end{cases} \tag{20}$$

The results are shown in Table 2 and Figure 2, which show that the fitting error is increased only slightly. However, the condition number is greatly improved, so that the results are much less sensitive to additional sampled engines than previously. The system for the remaining three fitting constants $W_{()}$ is now also linear, so that conventional linear regression could have been used to obtain them instead of the Newton method described here.

Table 2: Fit constants for turbofan engines. Condition number $C = 2213$.

b_m	b_π	b_α	W_0	W_π	W_α
1.00000	1.00000	1.20000	1684.5 lb	17.7 lb	1662.2 lb

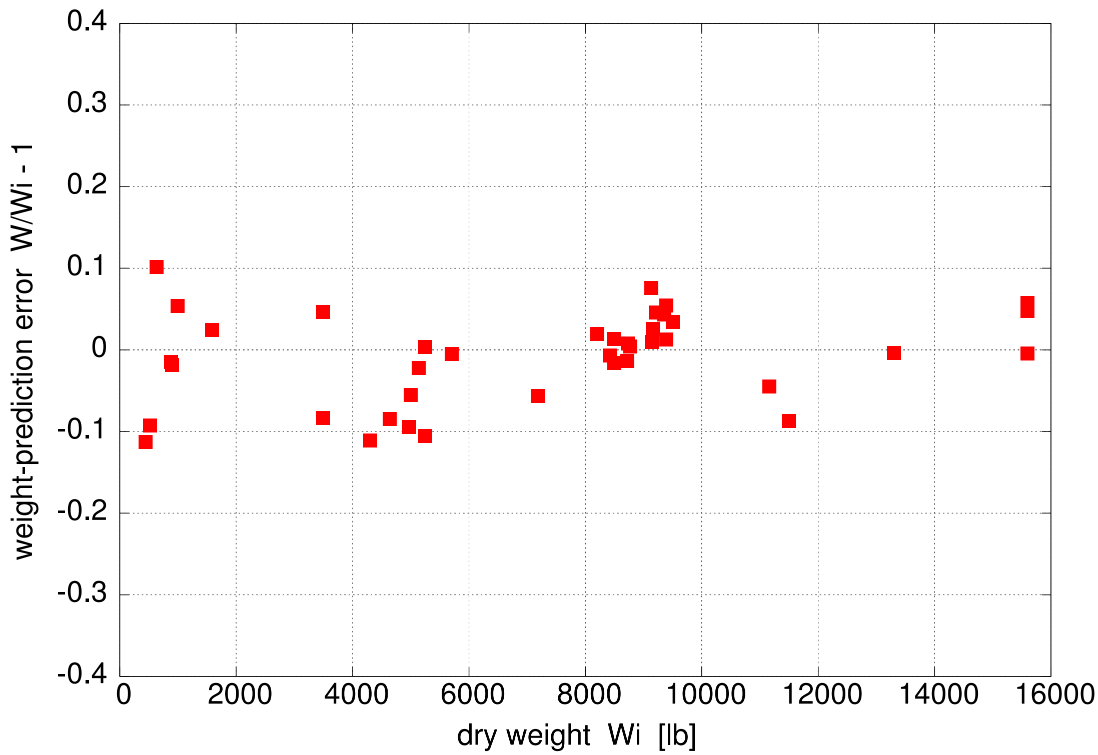

Figure 2: Weight-prediction error for sampled engines, versus engine weight, with frozen fitting exponents. Fitting error $\mathcal{E} = 0.0556$.

5 Nacelle weight model

A suitable weight model for the nacelle is given by Beltramo et at[2]. This is broken down into the inlet cowl between the lip and fan, the cowl around the fan, the exhaust cowl between the fan and the fan nozzle, and the core cowl.

$$W_{\text{nace}} = W_{\text{inlet}} + W_{\text{fan}} + W_{\text{exit}} + W_{\text{core}} \tag{21}$$

Each weight component is in turn correlated to its surface area and fan diameter as

$$W_{\text{inlet}} = A_{\text{inlet}} \, (2.5 + 0.0238 \, d_{\text{fan}}) \tag{22}$$

$$
\begin{aligned}
W_{\text{fan}} &= A_{\text{fan}}\, 1.9 && (23)\\
W_{\text{exit}} &= A_{\text{exit}}\, (2.5 + 0.0363\, d_{\text{fan}}) && (24)\\
W_{\text{core}} &= A_{\text{core}}\, 1.9 && (25)
\end{aligned}
$$

where the areas $A_{(\,)}$ are in square feet, d_{fan} is in inches, and the resulting weights are in pounds. A convenient way to specify the fan cowl area portions is as fractions of the overall nacelle area S_{nace}, which in turn is specified as some multiple $r_{S_{\text{nace}}}$ of the fan frontal area.

$$
\begin{aligned}
S_{\text{nace}} &= r_{S_{\text{nace}}}\, \pi (d_{\text{fan}}/2)^2 && (26)\\
A_{\text{inlet}} &= f_{\text{inlet}}\, S_{\text{nace}} && (27)\\
A_{\text{fan}} &= f_{\text{fan}}\, S_{\text{nace}} && (28)\\
A_{\text{exit}} &= f_{\text{exit}}\, S_{\text{nace}} && (29)
\end{aligned}
$$

The core cowl area is assumed to be some multiple of the LPC frontal area, specified by its diameter d_{LPC}.

$$
A_{\text{core}} = r_{\text{core}}\, \pi (d_{\text{LPC}}/2)^2 \qquad (30)
$$

Representative values for the ratios and fractions are listed below.

$$
\begin{aligned}
r_{S_{\text{nace}}} &= 12\\
f_{\text{inlet}} &= 0.4\\
f_{\text{fan}} &= 0.2\\
f_{\text{exit}} &= 0.4\\
r_{\text{core}} &= 12
\end{aligned}
$$

6 Accessory and pylon weight models

The added weight of the accessories is assumed to be some fraction of the engine bare weight,

$$
W_{\text{add}} = f_{\text{add}}\, W_{\text{bare}} \qquad (31)
$$

and the pylon weight is assumed to be proportional to the total weight mounted at the end of the pylon.

$$
W_{\text{pylon}} = f_{\text{pylon}} (W_{\text{bare}} + W_{\text{add}} + W_{\text{nace}}) \qquad (32)
$$

Suitable values for the fractions are listed below.

$$
\begin{aligned}
f_{\text{add}} &= 0.10\\
f_{\text{pylon}} &= 0.10
\end{aligned}
$$

The overall engine system weight is the sum of all the components defined above.

$$
W_{\text{eng}} = W_{\text{bare}} + W_{\text{add}} + W_{\text{nace}} + W_{\text{pylon}} \qquad (33)
$$

References

[1] `www.cfmaeroengines.com/engines/cfm56/`

[2] M.N. Beltramo, D.L. Trapp, B.W. Kimoto, and D.P. Marsh. Parametric study of transport aircraft systems cost and weight. Technical Report NASA CR 151970, NASA, April 1977.